SPECTRAL DOMAIN ANALYSIS OF DOMINANT AND HIGHER ORDER MODES IN FIN-LINES

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ABSTRACT

The spectral domain analysis is applied to deriving dispersion characteristics of the dominant and higher order modes in the fin-line structures. Numerical results are compared with those by a modified spectral domain technique as well as other data.

Introduction

The fin-line structure is a special printed transmission line developed for millimeter wave I. C. developed by Meier[1]. Propagation characteristics of the fin-line have been investigated by a number of workers such as Hofmann[2] and Hoefer[3]. In the former, two types of infinite summations appear in the numerical process and the truncation of these summations need to be done carefully in such a way that no relative convergence problem would arise. On the other hand, some engineering approximations are involved in the work in [3]. The present paper describes an application, to the fin-line structure, of the spectral domain technique developed for analysis of various printed transmission lines for microwave integrated circuits[4,5]. The main features of the present work are: (1) Accuracy of the numerical solutions obtained by the spectral domain method is checked against the data obtained by the modified version of the original spectral domain method. (2) In addition, we obtained dispersion curves for higher order modes. In practical applications, the knowledge of higher order modes is important because often it is necessary to find the frequency region in which the single mode operation is possible.

Theory

Since the details of the spectral domain method itself have been reported in [4] and [5], only the key steps will be given in the paper. The modified method to be used for accuracy check has recently been used for higher order mode analysis of microstrip lines[6]. The significance of the modifications will be pointed out in this paper.

Although the methods are applicable to other types of fin-line structures, we will formulate the problem for the bilateral fin-line, the cross-section of which is shown in Fig. 1 (a). Because of the symmetry, we only need to consider one-half of the structure given in Fig. 1 (b).

Since the modal field in the fin-line is of hybrid type, the fields in Regions 1 (dielectric) and 2 (air) can be derived from

$$E_{zi} = j\frac{k_i^2 - \beta^2}{\beta}\,\phi_i\,(x,y) \tag{1}$$

$$H_{zi} = j\frac{k_i^2 - \beta^2}{\beta}\,\psi_i\,(x,y) \tag{2}$$

where $i = 1,2$ signifies the region, k_i is the wave number in Region i and β is the propagation constant of the mode in the z direction. The time and z dependence of the field $\exp(j\omega t - j\beta z)$ is omitted throughout the paper.

In the spectral domain approach, the potentials ϕ_i and ψ_i as well as all the field quantities are Fourier transformed via

$$\tilde{\phi}_i(n,y) = \int_{-b}^{b} \phi_i(x,y)\, \exp(j\hat{k}_n x)dx \tag{3}$$

where $\hat{k}_n = \frac{n\pi}{b}$ for the dominant and all odd modes and $\hat{k}_n = (n - \frac{1}{2})\,\pi/b$ for the even modes. Because of the boundary conditions at $y = 0$ and a

$$\tilde{\phi}_1(n,y) = A_n^e \cosh \gamma_1 y, \qquad \tilde{\phi}_2(n,y) = B_n^e \sinh \gamma_2(a - y)$$

$$\tilde{\psi}_1(n,y) = A_n^h \sinh \gamma_1 y, \qquad \tilde{\psi}_2(n,y) = B_n^h \cosh \gamma_2(a - y)$$

where

$$\gamma_1 = \sqrt{\hat{k}_n^2 + \beta^2 - \varepsilon_r k_o^2}, \qquad \gamma_2 = \sqrt{\hat{k}_n^2 + \beta^2 - k_o^2}$$

and k_o is the free space wave number. We will now apply the interface conditions at $y = d$ to the appropriate field components. When this is done, the unknown coefficients A_n^e, A_n^h, B_n^e and B_n^h are eliminated and we obtain two coupled algebraic equations

$$Y_{xx}\tilde{E}_x + Y_{xz}\tilde{E}_z = \tilde{J}_x \tag{4}$$

$$Y_{zx}\tilde{E}_x + Y_{zz}\tilde{E}_z = \tilde{J}_z \tag{5}$$

where

$$Y_{xx} = (\varepsilon_r k_o^2 - \beta^2)\,\frac{\tanh \gamma_1 d}{\gamma_1} + (k_o^2 - \beta^2)\,\frac{\coth \gamma_2 h}{\gamma_2} \tag{6}$$

$$Y_{xz} = Y_{zx} = \beta\,\hat{k}_n\left(\frac{\tanh \gamma_1 d}{\gamma_1} + \frac{\coth \gamma_2 h}{\gamma_2}\right) \tag{7}$$

$$Y_{zz} = (\varepsilon_r k_o^2 - \hat{k}_n^2)\,\frac{\tanh \gamma_1 d}{\gamma_1} + (k_o^2 - \hat{k}_n^2)\,\frac{\coth \gamma_2 h}{\gamma_2} \tag{8}$$

$h = a - d$

are all known, and $\tilde{E}_x$, $\tilde{E}_z$ and $\tilde{J}_x$, $\tilde{J}_z$ are Fourier transforms of unknown tangential electric field in the gap ($y = d$, $|x| < s$) and unknown current components on the fins ($y = d$, $s < |x| < b$). Up to this stage the method of analysis is exact. In the following we present a solution based on the Galerkin's method.

To this end, the unknown aperture fields $\tilde{E}_x$ and $\tilde{E}_z$ are expanded in terms of known basis functions

$$\tilde{E}_x(\hat{k}_n) = \sum_{i=1}^{M} c_i\,\tilde{\xi}_i(\hat{k}_n) \tag{9}$$

$$\tilde{E}_z(\hat{k}_n) = \sum_{j=1}^{N} d_j \tilde{\eta}_j(\hat{k}_n) \qquad (10)$$

where the basis functions $\tilde{\xi}_i$ and $\tilde{\eta}_j$ are Fourier transforms of $\xi_i(x)$ and $\eta_j(x)$ which are chosen to be zero except for $|x| < s$.

Now (9) and (10) are substituted into (4) and (5) and the inner products of the resulting equations with each of $\tilde{\xi}_i$ and $\tilde{\eta}_j$ are obtained. The result is the homogeneous matrix equation for unknown c_i and d_j.

$$\sum_{i=1}^{M} K_{pi}^{xx} c_i + \sum_{j=1}^{N} K_{pj}^{xz} d_j = 0 \qquad p = 1,\ldots,M \qquad (11)$$

$$\sum_{i=1}^{M} K_{qi}^{zx} c_i + \sum_{j=1}^{N} K_{qj}^{zz} d_j = 0 \qquad p = 1,\ldots,N \qquad (12)$$

Equating the determinant of the coefficient matrix associated with (11) and (12) to zero, we obtain the eigenvalue equation and its solution gives the propagation constant β.

One of the features of the spectral domain method is that accurate solutions result even if we use an extremely small size matrix such as $M = N = 1$. This is because qualitative natures such as the edge condition of the aperture electric field can be incorporated in the choice of basis functions. In the present case we have set $M = N = 1$ and chosen $\tilde{\xi}_1$ and $\tilde{\eta}_1$ as the Fourier transforms of $\xi_1 = s/\sqrt{s^2 - x^2}$ and $\eta_1 = x\sqrt{s^2 - x^2}$ (see Fig. 2). Note that $\tilde{\xi}_1$ and $\tilde{\eta}_1$ are given analytically.

Modification Of Spectral Domain Method

Instead of checking the accuracy of the numerical solution by increasing the matrix size, we computed the propagation constant by use of the modified spectral domain method, which is a combination of the point matching and spectral domain method. In the modified method, the aperture electric fields are expressed in terms of trains of rectangular pulses with unknown amplitudes. For instance

$$E_x(x) = \sum_{i=1}^{M} \bar{c}_i G_i(x) \qquad (13)$$

$$E_z(x) = \sum_{j=1}^{M} \bar{d}_j G_j(x) \qquad (14)$$

where $G_i(x) = 1$ for $(i - 1)\Delta x < x < i\Delta x$, $\Delta x = 2s/M$ and zero elsewhere (see Fig. 2).

In the conventional point matching method, (13) and (14) are substituted into integral equations which are then discretized in observation space. We will not follow this approach. Instead we take Fourier transforms of (13) and (14), and then they are substituted into the algebraic equations (11) and (12) and proceed in the manner same as the spectral domain method. This modification has the following features:

(1) It is numerically less advantageous than the original spectral domain method, because it requires inherently larger size matrix.
(2) Edge conditions cannot be directly incorporated in the basis function.
(3) However, the new method is more flexible and the coefficients $\bar{c}_i$ and $\bar{d}_j$ are adjusted automatically to represent the aperture distributions. In the original method, the qualitative nature of the aperture field is fixed once the basis functions are selected.

Numerical Results

Dispersion characteristics of a bilateral fin-line have been computed by the spectral domain method. The results are plotted in Figs. 3 and 4. Note that the size of the shield case (a and b) coincides with that of the WR28 waveguide for 26.5 ∿40 GHz operation. The modified spectral domain method has been used to check the accuracy of the spectral domain data at several points on the graphs. Agreement seems quite satisfactory. The results also agree favorably with other available data.

It is clearly seen that the fin-line mode is not quasi-TEM, but rather it resembles that of the ridged waveguide as pointed out earlier[1,3].

In addition to the dominant mode, we found the first odd higher order mode. The modified spectral domain method also confirmed the existence of the higher order mode as seen in Figs. 3 and 4.

Conclusions

We presented solutions for dispersion characteristics of the fin-line, based on the spectral domain method and its modification. The accuracy of the solutions is checked by comparing the results by the two methods. The results for the higher order mode are also provided. Information such as the cutoff frequency of the higher order mode is believed useful in practice.

References

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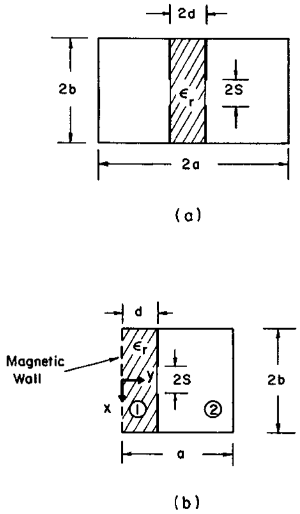

Fig. 1 Fin-line structure, (a) cross-section of the bilateral fin-line, (b) equivalent structure.

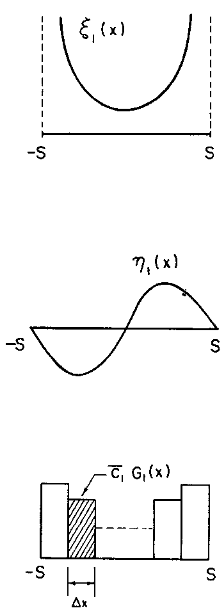

Fig. 2 Basis functions.

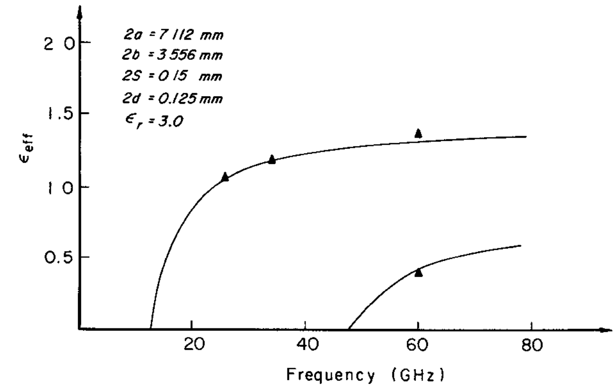

Fig. 3 Dispersion characteristics of fin-line modes; ——— spectral domain method, ▲ modified method.

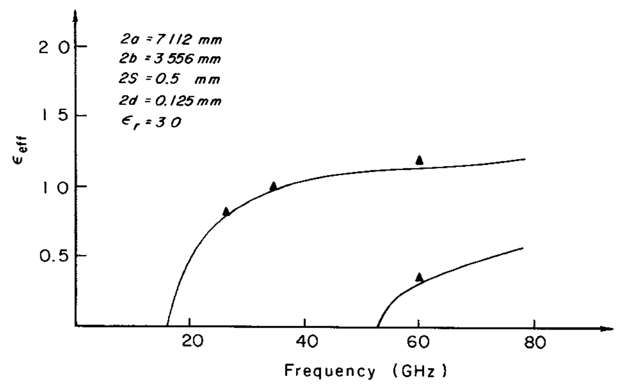

Fig. 4 Dispersion characteristics of fin-line modes; ——— spectral domain method, ▲ modified method.